A Penalty-Free Implicit Runge–Kutta Discontinuous Galerkin Method for Biharmonic equations

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ABSTRACT

We present a high-order implicit Runge–Kutta discontinuous Galerkin (RKDG) formulation for biharmonic type equations. The proposed method combines a penalty-free DG spatial discretization with an algebraically stable multi-stage implicit Runge–Kutta scheme, ensuring unconditional energy stability and arbitrary-order accuracy. This study provides a detailed error analysis quantifying temporal and spatial contributions to the total discretization error. Furthermore, a computational cost assessment demonstrates that the proposed RKDG approach achieves comparable or superior accuracy to traditional Crank–Nicolson DG schemes while significantly reducing CPU time for large time steps. Numerical experiments on benchmark problems confirm the theoretical error estimates, and computational efficiency of the all-at-once RKDG implementation.

Keywords

Runge–Kutta methods; Discontinuous Galerkin; Biharmonic problems; Gradient flows; Energy stability; Error analysis; Computational efficiency.