How to Combine Expert Estimates

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Formulation of the problem. In many practical situations, we rely on experts to estimate the probability of some future event E. To get a more accurate estimate, we ask several (n) experts and get their estimates $p_1, \ldots, p_n$. It is desirable to come up with a single estimate p that combines these estimates.

Reasonable assumption of independence. If the results of two experts are strongly correlated, it makes not much sense to ask both – because the probability provided by the second expert will be practically the same as the probability of the first expert. Thus, it makes sense to assume that the experts are independent.

Analysis of the problem and the resulting formula. To analyze this situation, let us consider a more general situation, when we have n events $E_1, \ldots, E_n$, and experts i estimates the probability of the i-th event E_i. In this case, we have 2^n possible situations: $E_1 \,\&\, \ldots \,\&\, E_{n-1} \,\&\, E_n$, $E_1 \,\&\, \ldots \,\&\, E_{n-1} \,\&\, \neg E_n$, $E_1 \,\&\, \ldots \,\&\, E_{n-1} \,\&\, \neg E_{n-1} \,\&\, E_n$, $\ldots$, $\neg E_1 \,\&\, \ldots \,\&\, \neg E_{n-1} \,\&\, E_n$. Since the experts are independent, the probability of each situation is equal to the product of the probabilities correpsonding to each E_i; $p_1 \cdot \ldots \cdot p_{n-1} \cdot p_n$, $p_1 \cdot \ldots \cdot p_{n-1} \cdot (1-p_n)$, $p_1 \cdot \ldots \cdot p_{n-2} \cdot (1-p_{n-1}) \cdot p_n$, $\ldots$, $(1-p_1) \cdot \ldots \cdot (1-p_{n-1}) \cdot (1-p_n)$.

In our case, when there is only one event, we cannot have this event both happening and no happening, we have only two possible situations: when E happens (with probability $p_1 \cdot \ldots \cdot p_{n-1} \cdot p_n$), and when E does not happen (with probability $(1-p_1) \cdot \ldots \cdot (1-p_{n-1}) \cdot (1-p_n)$); all other cases are inconsistent. Thus, under this condition of consistency, the (conditional) probability p that E will happen is equal to

$$p = \frac{p_1 \cdot \ldots \cdot p_{n-1} \cdot p_n}{p_1 \cdot \ldots \cdot p_{n-1} \cdot p_n + (1-p_1) \cdot \ldots \cdot (1-p_{n-1}) \cdot (1-p_n)}. \tag{1}$$

A natural question: when is the combined probability p larger than an individual estimate p_i – and is $p < p_i$? We show that this is equivalent to

$$p_1 \cdot \ldots \cdot p_{i-1} \cdot p_{i+1} \cdot \ldots p_n > (1-p_1) \cdot \ldots \cdot (1-p_{i-1}) \cdot (1-p_{i+1}) \cdot \ldots (1-p_n), \tag{2}$$

or, equivalently, as $o_1 \cdot \ldots \cdot o_{i-1} \cdot o_{i+1} \cdot \ldots o_n > 1$, where the odds o_j are defined as $o_j \stackrel{\text{def}}{=} p_j/(1-p_j)$.

In particular, for $n = 2$ and $i = 1$, this condition is equivalent to $p_2 > 1 - p_2$, i.e., equivalently, to $p_2 > 0.5$. This makes perfect sense: $p_2 > 0.5$ means that the second expert is more confident that E will happen than that it will not happen. This positive belief in E increase the coverall probability of E. Vice versa, if the second expert is more negative about E, this decreases our confidence that E will happen.

For $n = 3$ and $i = 1$, this inequality is equivalent to $(p_2 + p_3)/2 > 0.5$. In this case, our confidence increases if, on average, the other two experts believe in E more than in $\neg E$.

What if there is a small correlation r between experts. In this case, for $n = 2$, we get

$$p = \frac{p_1 \cdot p_2 + r \cdot s}{p_1 \cdot p_2 + r \cdot s + (1-p_1) \cdot (1-p_2) + r \cdot s}, \text{ where } s \stackrel{\text{def}}{=} \sqrt{p_1 \cdot p_2 \cdot (1-p_1) \cdot (1-p_2)}.$$