# Why biological neurons are most effective at the border of the set of possible states

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**Formulation of the problem.** Recent research shows that the brain's performance is the best when the neurons' states are the border of the set of possible states. How can we explain this phenomenon?

**Our explanation.** To explain this phenomenon, let us recall some basic facts from calculus. In general, the maximum of a continuous function $f(x_1, \ldots, x_n)$ on a closed domain $D$ is attained either at the interior of this domain or on the domain';s border. When the function is smooth, and its maximum is attained at the interior point, then, according to calculus, all partial derivatives of this function should be equal to 0:

$$\frac{\partial f}{\partial x_1} = \frac{\partial f}{\partial x_2} = \ldots = \frac{\partial f}{\partial x_n} = 0.$$

In other words, in this case, the point at which the maximum is attained is one of the stationary points of the function $f(x_1, \ldots, x_n)$.

A function usually has a small number of stationary points such as local and global minima and maxima. When the domain $D$ is small, the probability that this domain happens to contain one of these stationary points is small. So, in most cases, the maximum of a smooth function on a small domain is attained at the border of this domain.

Let us apply this general idea to our case. Neurons – like many other biological systems – are very vulnerable to outside changes: they can only operate within a small range of temperatures, and a small range of other parameters. Thus, the set $D$ of possible states is indeed very small in comparison with the set of all theoretically possible states – that not necessarily support the living cells. So, a natural conclusion is that the optimal behavior – i.e., the behavior for which the effectiveness $f(x_1, \ldots, x_n)$ attains its largest possible value – is indeed attained at the border of this set $D$.

*Comment.* This phenomenon is ubiquitous, it is not limited to the brain. For example, the optimal regime of a nuclear power station is when its state is at the edge of instability – when the reproduction rate of neurons is exactly 1. A more mundane example: when we walk at a usual speed, we lift a foot up and forward and thus, get an unstable state, on the edge of falling down – and we are saved by putting the foot down. The intermediate state is indeed unstable, it is easy to fall down if some obstacle is suddenly encountered when the foot is up.

A. Curatolo, "Brain response relies on avalanches", *APS Physics*, 2025, Vol. 18, Paper s4.

Z. Yang, J. Liang, and C. Zhou, "Critical avalanches in excitation-inhibition balanced networks reconcile response reliability with sensitivity for optimal neural representation", *Physical Review Letters*, 2025, Vol. 134, Paper 028401.