Laplacian Eigenvalues of Bipartite Kneser-Like Graphs

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Bipartite Kneser-like graphs $G(a, b)$ are defined by $a, b \in \mathbb{N}$ such that $a > b$ and letting $n = a + b + 1$. Then, let $\mathcal{A}$ and $\mathcal{B}$ be a-sized and b-sized subsets of $[n]$, respectively. An edge is drawn between all vertices of $A \in \mathcal{A}$ and $B \in \mathcal{B}$ if and only if $A \cap B = \emptyset$. The Laplacian matrix of $G(a, b)$ is defined as $L(G(a, b)) = Deg(G(a, b)) - Adj(G(a, b))$. A general construction of these graphs' eigenvectors has been proven by a previous student, and now we aim to prove that we can always construct a basis of vectors using this combinatorial description of the eigenvectors.

Using the general construction of the eigenvectors for these Kneser-like graphs we can generate a large but finite number of eigenvectors. We seek to use a lexicographic order on these eigenvectors to pick enough linearly independent eigenvectors to form a basis for $L(G(a, b))$ for any a and b. This would demonstrate that all Laplacian eigenvalues of these graphs are always non-negative integers.