Classification of 3-Regular Quantum Graphs in $M_3(\mathbb{C})$

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Quantum graphs are an analogue of graphs in the field of non-commutative geometry and have applications to error correction in quantum communication. A class of quantum graphs arises as subspaces of $M_n(\mathbb{C})$ which are closed under taking the conjugate transpose of every matrix in the subspace. We call such a quantum graph irreflexive if every matrix has trace 0, and two quantum graphs P and Q are said to be isomorphic if there exists a unitary matrix U so that $UPU^{-1} = Q$. A special type of quantum graph in $M_n(\mathbb{C})$, called a d-regular quantum graph, is a d-dimensional quantum graph $R \subseteq M_n(\mathbb{C})$ that is irreflexive and satisfies the property that for any Hermitian basis $\mathcal{B}$ of R, the sum of the squares of the matrices in $\mathcal{B}$ is a multiple of the identity matrix. We classified all of the 3-regular quantum graphs in $M_3(\mathbb{C})$ up to isomorphism, showing that every 3-regular quantum graph in $M_3(\mathbb{C})$ is isomorphic to a quantum graph of the form

$$R(\epsilon) = \mathbb{C}\begin{bmatrix} \epsilon & \sqrt{1-\epsilon^2} & 0 \\ \sqrt{1-\epsilon^2} & -\epsilon & 0 \\ 0 & 0 & 0 \end{bmatrix} + \mathbb{C}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} + \mathbb{C}\begin{bmatrix} 0 & 0 & i \\ 0 & 0 & 0 \\ -i & 0 & 0 \end{bmatrix}$$

for some $\epsilon \in [0,1]$ and that any two distinct choices of ϵ in $[0,1]$ give non-isomorphic quantum graphs.