Dispersion Relation Preserving High-order Finite Difference method for Numerical Simulation of Acoustic Waves

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Abstract

Accurate simulation of acoustic wave propagation requires numerical methods that are both stable and capable of controlling dispersion errors over long propagation distances. In this work, we study high-order finite-difference discretizations for the three-dimensional acoustic pressure–velocity system implemented in WaveQLab3D. We compare traditional summation-by-parts (SBP) operators, dual-pair SBP (DP-SBP) operators of fourth and sixth order, and dispersion-relation-preserving (DRP) operators of fourth and sixth order. Simultaneous approximation terms (SATs) are used to impose boundary conditions, and energy estimates are derived to establish the stability properties of the resulting semi-discrete SBP-SAT formulations. Numerical simulations are performed on the UTEP/Jakar high-performance computing system using a sequence of grids ranging from 101^3 to 301^3. The methods are compared through receiver-based pressure signals, numerical errors, grid-refinement behavior, and dispersion analysis. The results show clear differences among the schemes in phase accuracy and error behavior, with the DRP formulations providing improved dispersion properties over a broader range of resolved wave numbers while retaining the SBP structure required for stable computations. These results provide a basis for further development of accurate and stable acoustic-wave simulation methods.

Keywords: Acoustic wave propagation; DP-SBP-SAT methods; Dispersion-relation-preserving methods; Numerical stability..