Temperature-Dependent Multifractal Signatures of Criticality in the Two-Dimensional Ising Model

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Abstract

This study investigates how critical behavior in the two-dimensional Ising model is reflected in the multifractal properties of magnetization dynamics. Equilibrium magnetization time series were generated over a range of temperatures spanning the ordered, critical, and disordered regimes using Metropolis Monte Carlo simulations. Magnetic susceptibility was used to characterize fluctuations around the critical region. Multifractal detrended fluctuation analysis (MFDFA) was then applied to the magnetization series and their first-order increments to examine temperature-dependent changes in scaling behavior. The analysis considered generalized Hurst exponents, mass exponents, multifractal spectra, spectrum width, and related multifractal descriptors. The results reveal distinct changes in temporal persistence and multifractal structure across temperature. Magnetization exhibited strong persistence near the critical region, while its increments displayed weaker dependence and more stationary behavior. Multifractal measures also varied with temperature, suggesting that the transition from ordered to disordered states leaves measurable signatures in magnetization dynamics. These findings motivate further investigation of the relationship between multifractality, critical phenomena, and renormalization-group scaling.

Keywords: Two-dimensional Ising model; criticality; phase transition; multifractal analysis; MFDFA; magnetization; Monte Carlo simulation