Gain Probability for Real-Valued Distributions: Theory and Applications in Finance and Economics

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Abstract

Gain-probability (GP) analysis quantifies the probability that a randomly selected outcome from one group exceeds a randomly selected outcome from another, providing a probabilistic comparison based on the full distributions rather than solely on differences in location. Existing GP frameworks are largely restricted to nonnegative distributions or same-family pairings, limiting their applicability to outcomes that take both negative and positive values, such as log-returns, log-wage differences, and profit-and-loss measures. This paper develops a unified GP framework for real-valued outcomes, defined as $GP_{a,c}(X,Y) = P(X + aY \leq c)$, for four continuous distribution families: Skew-Normal, Skew-(t), Laplace, and Logistic. A general convolution theorem expresses GP for any cross-family pairing as a one-dimensional integral, while exact or semi-closed-form results are derived for same-family pairings. To facilitate practical implementation, a three-step model-selection workflow is developed and an interactive R Shiny application is provided for GP computation. Two empirical applications demonstrate the framework in finance and economics: comparing daily log-returns of the DAX and CAC 40 European stock indices, and comparing log-wages by gender in the U.S. labour market. The proposed framework extends GP analysis to a broader class of real-valued outcomes and provides a flexible approach to probabilistic comparison when the distributions of the groups may differ in location, scale, skewness, or tail behaviour.

Keywords: Gain probability; Skew-Normal; Skew-t; Laplace; Logistic

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