Mechanical Semantics of Graded Safety: Trusses, Load Paths, and Which Connective Appears

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Abstract. Numbers in $[0, 1]$ arise naturally as normalised safety margins, probabilities, and resilience indices. Their common numerical range, however, does not justify applying the same composition rule to all of them. We investigate how many-valued operations acquire a mechanical interpretation when the graded predicate, physical operation, and admissible domain are specified.

For a member carrying signed axial force N_i, we define its remaining stress-reserve grade by

$$S_i = \left[1 - \frac{|N_i|}{A_i \sigma_{\max,i}}\right]_+, \qquad S_{\text{sys}} = \min_i S_i, \qquad [z]_+ = \max(0, z),$$

where A_i is cross-sectional area and $\sigma_{\max,i}$ is allowable stress magnitude. Minimum combines member predicates in one loaded configuration. A different question is whether the system grades of two separate load cases determine the grade when those loads act together.

Step-by-step examples, from a stepped bar to larger trusses, show that load paths determine the answer. Disjoint paths yield minimum, reinforcing shared paths yield Łukasiewicz conjunction, and opposing sublimit loads can yield Łukasiewicz equivalence. Partially shared paths produce weighted operations; some mechanically exact rules are noncommutative or nonassociative and therefore are not t-norms. For fixed linear member-force models, symmetry of the admissible load set about both load-coordinate axes characterises when an exact rule exists for all load signs. When signs or overload magnitudes are discarded, identical input grades can require different outputs, ruling out a universal scalar rule.

Further examples distinguish nonlinear response, probabilistic intersection, and time-averaged resilience. Independent member-survival events give the product t-norm, whereas instantaneous minimum functionality does not generally induce minimum composition of resilience averages. The missing information may concern load direction, geometric branch, or temporal overlap.

These results provide mechanical benchmarks for interpreted many-valued operations. They show why an applied connective must be derived or validated against the underlying phenomenon: sometimes a familiar operator is exact, sometimes a more general operation is required, and sometimes the graded representation must retain additional state information.

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