Strings and scale-invariance – why in string theory, amplitudes decrease faster than polynomials: a qualitative explanation

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In traditional field theory, elementary particles are point-wise. According to modern physics, the world consists of elementary particles – i.e., particles that cannot be divided into somewhat independent parts. According to relativity theory, the speeds v of all communications are bounded by the speed of light c: otherwise, $v > c$ will lead to sending information to the past, i.e., to causality violations. Thus, if an elementary particle was not point-wise, several spatial points from this particle would not be able to immediately communicate with each other and coordinate their movements – and thus, would be somewhat independent. This would contradict the definition of an elementary particle. Thus, elementary particles should be point-wise objects.

Point-wise particles lead to physically meaningless infinities. This sounds reasonable, but it leads to physically meaningless infinities. For example, what is the overall energy of the electric field $\vec{E}(x)$ generated by a charged particle? According to Coulomb's law, $|\vec{E}(x)| = c_0 \cdot \dfrac{q}{r^2}$ for some constant c_0, where q is the particle's charge and r is the distance to the particle. The energy density $\rho(x)$ is proportional to $|\vec{E}(x)|^2$, so $\rho(x) \sim r^{-4}$. Integrating $\rho(x)$ over the whole space leads to infinity: $\int r^{-4}\,dx = \int_0^\infty r^{-4} \cdot 4\pi \cdot r^2\,dr = 4\pi \cdot \int_0^\infty r^{-2}\,dr = -4\pi \cdot r^{-1}|_0^\infty = \infty$.

This is not only about Coulomb law. In general, a point-wise particle is invariant with respect to rotations and scaling $x \to \lambda \cdot x$, so it makes sense to assume that all related physical quantities q are also rotation- and scale-invariant, i.e., that $q(x)$ depends only on r ($q(x) = q(r)$) and that $q(\lambda \cdot r)$ is the same as $q(r)$ – modulo maybe appropriate re-scaling of q: $q(\lambda \cdot r) = c(\lambda) \cdot q(r)$ for some re-scaling parameter $c(\lambda)$. It is known that all measurable solutions to this equations are $q(r) = A \cdot r^\alpha$. (In particular, amplitudes decrease as power law for large r.) The problem is that for each α, the integral of $q(r)$ over the whole space has a physically meaningless infinite value.

Enter string theory. The only way to avoid infinities is to consider non-point-wise particles, of which connected linear-shaped strings are a simple example. (By the way, this does lead to local – and potentially global – causality violations.) These particles have a fixed length – e.g., their diameter – so the corresponding dependencies are no longer scale-invariant and can, therefore, differ from power laws.

Why faster than power law? In string theory, amplitudes decrease faster than power law. Why faster? Here, we have the joint effect of several parts that are not fully coherent. Coherence usually increases the overall effect, so its lack decreases the overall effect.