Why Are Egyptian Pyramids So Stable?

Tanzim Ahad[1], Victor M. Diaz[1], Glenn Duran[1], Md Tausif Al Hossain[1],
Leobardo Valera[2], and Vladik Kreinovich[1]
[1]Department of Computer Science, University of Texas at El Paso
500 W. University, El Paso, Texas 79968, USA
tahad@miners.utep.edu, vmdiaz6@miners.utep.edu,
gduran8@miners.utep.edu, mhossain15@miners.utep.edu, vladik@utep.edu
[2]Computer Science and Information Technology Systems Department
El Paso Community College, 919 Hunter, El Paso, TX 79915, USA
leobardovalera@gmail.com

Formulation of the problem. Egyptian pyramids have shown a lot of stability. Most of them have survived for millenia in spite of the periodic earthquake, sand storms, and – for many centuries – lack of maintenance. Why are they so stable?

A recent study used complicated mathematical analysis to explain the stability of the most well-known Khufu Pyramid (also known as the Pyramid of Cheops). The corresponding calculations are very specific, they only apply to this specific pyramid, but other pyramis are also stable. It is therefore desirable to have a more general explanation of stability, an explanation that is applicable to all ancient pyramids of Egypt.

Analysis of the problem. For convenience, let us start the coordinate system at the top point of a structure, and let the vertical coordinate z go down – so that all z-values $z = f(x, y)$ are non-negative.

Egyptian pyramids stand in the middle of the desert. With time, the bottom part of any building gets covered by sand, so what remains visible is only the upper part. The sand-covered part is definitely stable, so the question becomes – how stable is the visible upper part. Instead of the original shape $z = f(x, y)$, we have a new shape described by the same function $z = f(x, y)$ but limited to the values (x, y) for which $z(x, y) \le z_0$ for an appropriate z_0. An *alternative* is thus a family consisting of the original shape and all sand-covered shapes.

Whatever we optimize – stability or something else – we have a transitive relation $a \succeq b$ meaning that a is better than b (in the sense of the selected criterion). If there are several best alternatives, this means that our optimality criterion is not final: we can use this non-uniqueness to optimize something else. For example, if several shapes are most stable against earthquakes, we select among them the shape that is most stable against sandstorms. So, for a final criterion, there is only one optimal alternative

Most mechanical processes are invariant with respect to scaling $x, y, z \to x' = c \cdot x, y' = c \cdot y, z' = c \cdot z$. This is the reason why studying airplane scaled-down models in a wind tunnel provides a good understanding of the behavior of the actual planes. In general, if a final optimality criterion is invariant under some transformation g – i.e., if $a \succeq b \Leftrightarrow g(a) \succeq g(b)$ – then the optimal alternative $a_{\rm opt}$ is also invariant under g: $g(a_{\rm opt}) = a_{\rm opt}$.

In our case, invariance of the family of shapes means that re-scaling of the original shape should lead to one of the sand-covered shapes from the optimal family. If sand cover reduced the height from H to $h = c \cdot H$, then the remaining part $z = f(x, y)$ for $z \le h$, should be the same as the c-scaled original one: $z' = f(x', y')$. Substituting expressions for x', y', and z' into this formula, we get $c \cdot z = f(c \cdot x, c \cdot y)$. Since $z = f(x, y)$, we get $f(c \cdot x, c \cdot y) = c \cdot f(x, y)$. In particular, along each straight line $y = k \cdot x$ passing through 0 in the xy-plane, we have $f(c \cdot x, c \cdot k \cdot x) = c \cdot f(x, k \cdot x)$. In particular, for $c = X$ and $x = 1$, we get $f(X, k \cdot X) = f(1, k) \cdot X$. This is a linear dependence, so the corresponding line in the shape is a straight line coming from 0. And this is exactly the definition of a pyramid.