Motion of DNA Molecules in Cancer Cells: A Mathematical Explanation of the Empirical Phenomenon

Esteban Diaz[1], Oscar Medina[1], Nathaniel J. Santillan Romero[1],
Farjana Akter Sumi[1], Leobardo Valera[2], and Vladik Kreinovich[1]
[1]Department of Computer Science, University of Texas at El Paso
500 W. University, El Paso, Texas 79968, USA
njsantillan@miners.utep.edu, fasumi@miners.utep.edu,
omedina5@miners.utep.edu, ediaz30@miners.utep.edu, vladik@utep.edu
[2]Computer Science and Information Technology Systems Department
El Paso Community College, 919 Hunter, El Paso, TX 79915, USA
leobardovalera@gmail.com

Formulation of the problem. One-cell organisms multiply as much as they can – this is how they survive and thrive. For cells in a multi-cell organism – e.g., in our body – cells reproduction is restricted by the need to stay together. Sometimes, a mutation weakens the mechanisms for such restrictions, and the cells start reproducing uncontrollably. This is what cancer is about.

DNA molecules start actively moving when the cell is about to reproduce. It is therefore expected that the analysis of such movement in cancer cells can help to detect the reproduction (tumor growth) stage before the growth starts – and thus, hopefully, develop techniques for preventing this growth before it starts. For this purpose, research study the motion of DNA molecules in cancer cells.

A recent study analyzed how this motion changes with time. It turns out that when the motion starts, it follows, in the first approximation, a faster power law, but when the motion decreases, it follows a slower exponential decrease. This asymmetry makes cells even more prone to reproduce and thus, make cancer ever more dangerous. A natural question is: how can we explain this asymmetry?

Main ideas behind our explanation. In most physical processes, there is either no special moment of time, or no special unit of time. In the first case, the process does not change if we start our analysis at a different moment, i.e., change from t to $t+s$ for some constant s. This change may require a corresponding re-scaling of the observed quantity x, from x to $c \cdot x$ for some constant c depending on s. So we have $x(t+s) = c(s) \cdot x(t)$. It is known that the only continuous solution to this equation is $x(t) = A \cdot \exp(a \cdot t)$ for some constants A and a. Indeed, if we differentiate this equation with respect to s and set $s = 0$, we get $\frac{dx}{dt} = a \cdot x$, where $a \stackrel{\text{def}}{=} c'(0)$. Separating variables, we get $\frac{dx}{x} = a \cdot dt$. Integrating, we get $\ln(x) = a \cdot t + c_1$, hence $x(t) = C \cdot \exp(a \cdot t)$, where $C \stackrel{\text{def}}{=} \exp(c_1)$.

In the second case, we similarly have $x(\lambda \cdot t) = c(\lambda) \cdot x(t)$. Differentiating with respect to λ and plugging in $\lambda = 1$, we get $t \cdot \frac{dx}{x} = a \cdot dt$, where $a \stackrel{\text{def}}{=} c'(1)$. Separating variables, we get $\frac{dx}{x} = a \cdot \frac{dt}{t}$. Integrating, we get $\ln(x) = a \cdot \ln(t) + c_1$ hence we get power law $x(t) = A \cdot t^a$, where $A \stackrel{\text{def}}{=} \exp(c_1)$.

Resulting explanation. Let us start our measuring time when the motion starts increasing or decreasing – which will then be moment 0. When the motion starts, we have $x(0) = 0$. Out of the two models, only the power law has a moment when x is 0, so the process follows the power law. On the other hand, we cannot have power law describing decrease: in this case, $a < 0$, so we at moment 0, we would have $x = \infty$. Thus, for decrease, we have an exponential law – exactly as observed.