Optimal weight-loss diet

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Formulation of the problem. A recent study showed that the best diet to lose weight is routinized eating, when a person eats the same things every day, while the worst way is a cyclic yo-yo diet, when each cycle includes a starving period followed by an eating period. How can we explain this?

Optimality and invariance: reminder. A general optimality criterion means that we have a way to compare two alternatives. In precise terms, this means that on the set A of all alternatives, we have a binary relation $a \succeq b$ meaning that a is better or of the same quality as b. Naturally this relation must be transitive: if a is better than b and b is better than c, then a is better than c. If there are several best alternatives, this means that our criterion is not final: we can use this non-uniqueness to optimize something else. Thus, for a final optimality criterion, there should be exactly one optimal alternative. We can use this as a definition of final criterion.

Often, there are some reversible transformations $g : A \to A$ that should not change which alternative is better, i.e., for which $a \succeq b \Leftrightarrow g(a) \succeq g(b)$. For example, the comparison of different temperatures should not change if we use a different measuring unit and a different starting point – which corresponds to the transformation $g(x) = a \cdot x + b$.

One can prove that for each g-invariant final optimality criterion $\succeq$, the optimal alternative a_{opt} is itself invariant, i.e., $g(a_{\text{opt}}) = a_{\text{opt}}$. Indeed, since a_{opt} is optimal, we have $a_{\text{opt}} \succeq a$ for all $a \in A$. In particular, $a_{\text{opt}} \succeq g^{-1}(a)$ for all $a \in A$, where g^{-1}, as usual, means the inverse transformation. Due to g-invariance of $\succeq$, we can conclude that $g(a_{\text{opt}}) \succeq g(g^{-1}(a)) = a$. Thus, the alternative $g(a_{\text{opt}})$ is optimal. But since the criterion $\succeq$ is final, there is only one optimal alternative, so indeed $g(a_{\text{opt}}) = a_{\text{opt}}$.

Let us apply this idea to our case. People's biological processes do not change much from one day to another, i.e., relative to a shift in time by 24 hours. It is therefore reasonable to conclude that the relative quality of different diets – not necessarily cyclical – should not change under such shift. Thus, it makes sense to conclude that the optimal diet should also be similarly invariant – i.e., that in the optimal weight-loss diet, people should eat the same food day after day. This is exactly what experiments show.

How to explain the worst diet. According to calculus, the minimum of a function on any domain is attained either at the point where all derivatives are 0 or at the boundary of the domain. Stationary points are rare, so in most cases, the minimum is attained at the boundary. In our case, the domain is that calorie input on each day is non-negative, so the boundary is when it is 0. We cannot have all 0s – a person will die – so we get a diet in which almost-no-food periods alternate with eating periods. This is exactly the yo-yo diet – shown to be the worst.