What information about gas emissions is most helpful in earthquake predictions: theoretical explanation of an empirical fact

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Formulation of the problem. On the microscopic level, rocks are not perfectly solid. They contain small holes and spaces in which gases such as radon, helium, argon, etc. are often trapped. When a rock develops cracks, some of these gases are released. The release of gases is a good indication that the rock will break. This is a known phenomenon: e.g., earthquakes are often preceded by emission of naturally occurring gases.

It is therefore desirable to use the measurements of these gas releases to predict when and where the rocks will break – and thus, e.g., when and where the earthquake will occur. Researchers have been trying to make these predictions based on the measured gas signal $x(t)$. Until recently, these predictions were not very successful, since a lot of information in the signal is caused by some unrelated random phenomena. A recent paper showed that we can get much better predictions if instead of the whole signal $x(t)$, we use only two values: its maximum value $x_{\max}$ and its value x_{eq} when the signal reaches its equilibrium.

How can we explain why these two values are most important?

Our explanation. We want to know which values of the signal $x(t)$ most contribute to the overall effect. At each moment t, the gas release $x(t)$ is determined by the breaking rate r at this moment: $x(t) = f(r(t))$, for some increasing function $f(x)$. Thus, $r(t) = g(x(t))$, where $g(x)$ is the inverse function to $f(x)$. The overall breaking degree b can be determined by integrating the breaking rate: $b = \int r(t)\,dt = \int g(x(t))\,dt$.

We want to know which values of the signal x contribute the most. To answer this question, we should use x – and not t – as the variable. By definition of the derivative $x'(t)$, $dx = x'(t) \cdot dt$, thus $dt = dx/x'(t)$, and the expression for b takes the form $b = \int \frac{g(x)}{x'(t)}\,dx$. According to this formula, the largest contribution comes from the values x for which the integrated expression is the largest. This expression is the largest when the derivative is the smallest – namely, when it is equal to 0.

According to calculus, the derivative is 0 at the point where a function attains its maximum – and also at the equilibrium point, where the function stops changing. This explains the empirical observation.

This explanation can be viewed as a particular case of invariance-optimality relation. In general, for an invariant final optimality criterion, the optimal alternative is also invariant. In our case, we have a natural shift-invariance $t \to t + \Delta t$, so it makes sense that the most important values are the ones that are shift-invariant, i.e., for which $x(t + \Delta t) \approx x(t)$, i.e., for which $x(t + \Delta t) - x(t) \approx 0$. Since $x(t + \Delta t) - x(t) \approx x'(t) \cdot \Delta t$, this means selecting values at which $x'(t) = 0$ – exactly what was observed.