LUCAS SEQUENCES AND DIRICHLET INVERSES

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Multiplicative arithmetic functions form an abelian group under the operation of Dirichlet convolution.We can construct a prime-independent multiplicative function f from an integer sequence $\{X_n\}_{n\geq 1}$ (where $X_1 = 1$) by defining $f(p^n) = X_{n+1}$ (p being an arbitrary prime and $n \geq 0$). When $\{X_n\}_{n\geq 1}$ is a Lucas sequence of the first kind, its corresponding function f exhibits unique properties:

- Its Dirichlet inverse f^{-1} vanishes for higher powers, meaning: $f^{-1}(p^n) = 0$ whenever $n > 2$.
- The sequence $\{X_n\}_{n\geq 1}$ is Lucas if and only if f is specially (quadratic) multiplicative.

These functions are characterized by Busche-Ramanujan identities, which ultimately describe the fundamental identities of Lucas numbers. Furthermore, this framework allows us to explore the mathematical connections between right-shifted Lucas sequences and rational arithmetic functions.